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Soft triaxial rotor in the vicinity of $\gamma = \pi/6$ and its extensions

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Abstract. The collective Bohr Hamiltonian is solved for the soft triaxial rotor around $\gamma_0 = \pi/6$ with a displaced harmonic oscillator potential in γ and a Kratzer-like potential in β. The properties of the spectrum are outlined and a generalization for the more general triaxial case with $0 < \gamma < \pi/6$ is proposed.

PACS. 21.60.Ev Collective models – 21.10.Re Collective levels

Analytic or approximated solutions of the Bohr collective model may be given for a variety of different model potentials. The functional dependence of this potential on the deformation (β) and asymmetry (γ) variables determines the properties of the spectrum and eigenfunctions. These solutions are not limited to rigid cases (where either one or two of the variables are constrained to take a fixed value), but may be found in the case of soft potentials too (here the potential function is represented by a well and it is associated with extended wave functions). A soft solution represents a more physical case than a rigid one and a mathematical benchmark for our understanding of collective states in nuclear spectroscopy. Recently, Iachello introduced new solutions based on the infinite square well potential, named $E(5)$, $X(5)$ and $Y(5)$, to describe the critical point of shape phase transitions [1]. These solutions have initiated on one side intense and successful efforts aimed at the identification of the predicted patterns (nuclear spectra and electromagnetic properties) in experimentally observed spectroscopic data. On the other side a number of theoretical studies have explored new analytic solutions in various cases, from γ-unstable to axial rotor [2].

A solution of the stationary Schrödinger equation

$$H_{\mathrm{B}}\Psi(\beta,\gamma,\theta_i) = E\Psi(\beta,\gamma,\theta_i), \tag{1}$$

for the Bohr collective Hamiltonian, $H_{\mathrm{B}} = T_\beta + T_\gamma + T_{\mathrm{rot}} + V(\beta,\gamma)$ may be achieved for the β-soft, γ-soft triaxial rotor making use of a harmonic potential in γ and Coulomb-like and Kratzer-like potentials in β (see fig. 1):

$$V(\beta,\gamma) = V_1(\beta) + \frac{V_2(\gamma)}{\beta^2}\,, \tag{2}$$

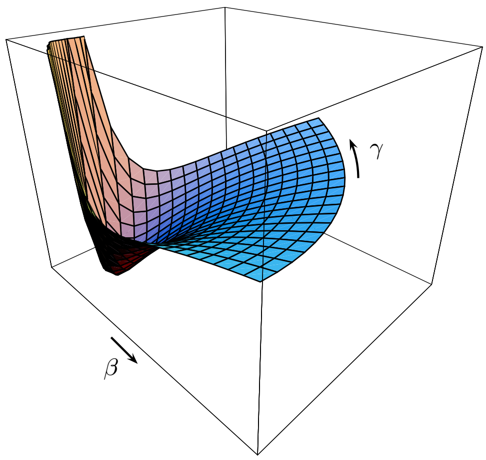

Fig. 1. Polar plot of the potential $V(\beta,\gamma)$ discussed in the text with minimum in $\gamma_0 = \pi/6$ and $\beta = 0.2$.

with

$$V_1(\beta) = -\frac{A}{\beta} + \frac{B}{\beta^2}\,, \qquad V_2(\gamma) = C(\gamma-\gamma_0)^2. \tag{3}$$

Unimportant multiplicative factors have been omitted here for simplicity. The Schrödinger equation above, (1), with the choice (2), is separable and can be solved in the vicinity of $\gamma_0 = \pi/6$, thus providing a paradigm for the spectrum of soft triaxial rotors.

It has been shown in [3] that the γ-angular part in the present case gives rise to a straightforward extension of the rigid triaxial rotor energy, also called Meyer-ter-Vehn formula [4], in which now an additive harmonic term appears, namely

$$\omega_{L,R,n_\gamma} = \sqrt{C}(2n_\gamma+1) + L(L+1) - \frac{3}{4}R^2\,, \tag{4}$$

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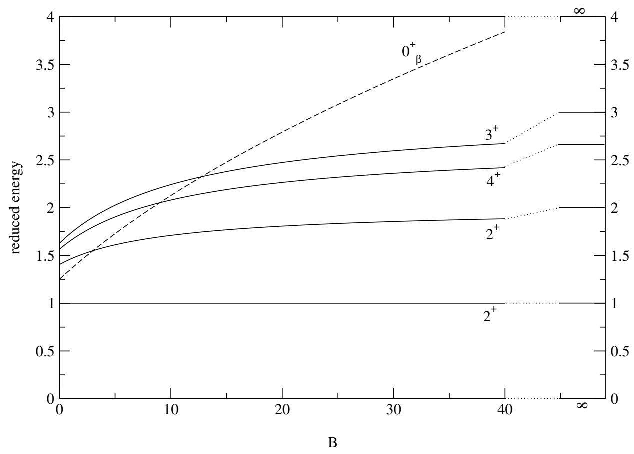

Fig. 2. Reduced energies of the lowest state of the β-band (dashed line) and of a few lowest states of the ground-state band (solid lines) as a function of B. The limits for the energy levels when $B \to \infty$, that correspond to the rigid triaxial rotor energies, are reported on the right side. Here we fixed $C = 1$. From [3].

where R is the quantum number associated with the projection of the angular momentum on the intrinsic 1-axis (that is a good quantum number for the $\gamma = \pi/6$ rotor [4]).

The solution of the equation in β depends on the particular choice of the β-potential and may results instead in a non-trivial expression for the energy spectrum. Using the Kratzer-like potential we obtain:

$$\epsilon(n_\gamma, n_\beta, L, R) = \frac{A^2/4}{(\sqrt{9/4+B+\omega_{L,R,n_\gamma}}+1/2+n_\beta)^2}\,. \quad (5)$$

The negative anharmonicities of the energy levels with respect to a simple rigid model are in qualitative agreement with general trends as observed in experimental data. This model is more general than the Davydov (rigid) model [5]: in fact the rigid model is recovered when the potential well becomes very narrow (that is when $B \to \infty$) as can be seen on the right side of fig. 2.

Here we present the expression of the spectrum in another well-known solvable case: the Davidson potential, $A_{\rm D}\beta^2 + B_{\rm D}/\beta^2$, discussed in [6,7] and references therein. The spectrum may again be found in an analytical way. We obtain

$$\epsilon_{\rm D}(n_\gamma, n_\beta, L, R) = \sqrt{A_{\rm D}}\left(2n_\beta + \tau_{L,R,n_\gamma} + 5/2\right), \quad (6)$$

where τ is found from $(\tau+1)(\tau+2) = B_{\rm D} - \omega_{L,K,n_\gamma}$.

Recently it has become possible to extend these results to soft triaxial rotors with a harmonic potential (as in eq. (3) on the right) centered around any asymmetry in the sector $0 < \gamma_0 < \pi/3$ by means of a group theoretical approach based on the $su(1,1)$ algebra [7]. Here the labeling is more difficult since neither K nor R (quantum numbers associated with the projections of the third component of the angular momentum on the 3rd and 1st intrinsic axis, respectively) are good quantum numbers, but a classification of the states is still possible on the basis of the remaining quantum numbers.

Retaining the same procedure used in the $\gamma_0 = \pi/6$ case for the separation of variables we are faced with the problem of solving the equation in γ that contains a rather complicated rotational kinetic term (Here a simplification like the one used in [1] (2nd paper) or [3] may not be adopted). The components of the moment of inertia that occur in that term are simplified here, neglecting fluctuations in the γ-variable, in the following way

$$A_\kappa = \frac{1}{4\sin^2(\gamma - 2\pi\kappa/3)} \longrightarrow \frac{1}{4\sin^2(\gamma_0 - 2\pi\kappa/3)}\,. \quad (7)$$

The equation in γ is then transformed in a set of coupled differential equations by expanding the (general triaxial) wave functions in a basis of rotational (axial) wave functions. Introducing also some standard trigonometric approximations it is possible to define a realization of the algebra $su(1,1)$ in terms of differential operators with which, for each L, we can reduce the secular problem to an algebraic equation. When the algebraic equation has a low order it can be solved analytically, while for higher orders one can always get a numerical (accurate) solution. The results have the same structure of eq. (4): the "rotational part", that coincides in every detail with the well-known solution of the rigid model, is accompanied by an additive harmonic term, that takes into account the γ quanta.

Once ω is obtained, it must be used in the differential equation in β, that may be solved in standard ways, depending again on the β-potential. This extension automatically generates the particular results obtained above when $\gamma_0 = \pi/6$.

This model contains in total 3 parameters (2 from the β and γ potentials, B and C, and one from the moments of inertia, A_3, or alternatively γ_0) and may provide a simple model for the interpretation of collective spectra of a large number of nuclei that do not posses axial symmetry. The dependence of the reduced spectrum on the three parameters is however non-linear and at present we have only applied the model to spectroscopic data in a preliminary way. A more complete description of the problem, of the methodology used and applications will soon be presented [7].

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